

Mathematics 551 Homework, April 8, 2020

First here is the basic template you will for this homework:

```
\documentclass[12pt]{amsart} % Use the AMS article class 12 point type size

\begin{document} % You put your input between these two

\end{document}
```

If you type:

Here are the basics of using LaTeX to typeset text (with no mathematics). You just type in the text and LaTeX will take care of the spacing, breaks and the like.

To start a new paragraph just skip one or more lines.
You can `\emph{emphasize`
one or more words} or make them `\textbf{bold}`.
It is even possible to
`\underline{underline stuff}`.

You will get:

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To start a new paragraph just skip one or more lines. You can *emphasize one or more words* or make them **bold**. It is even possible to underline stuff.

Typing

When type setting mathematics in a paragraph you change to `\emph{math mode}`. This is done putting the formula, either between two dollar signs (that is `\$ \dots \$`) or between `\verb+(+ and \verb+)+`. Thus

The powers of x are x^0 , x^1 , x^2 , x^3 , \ldots

A fraction looks like $\frac{1}{2}$

To the best of my knowledge Issac Newton was the first person to use fractional exponents to denote roots, that is the first to write such statements as $\sqrt{a} = a^{\frac{1}{2}}$ and $\sqrt[3]{b} = b^{\frac{1}{3}}$

The notation $\int_a^b f(x)dx$ for the integral is due to Leibniz. The sigma notation for a sum $\sum_{n=5}^{100} a_n$ is due to Euler.

I personally prefer the \dots version, but you use what you prefer.

To get greek letters in math mode you just use their Endlish names precided by a backslash: α , β , γ etc.

gives

When type setting mathematics in a paragraph you change to *math mode*. This is done putting the formula, either between two dollar signs (that is \dots) or between $($ and $)$. Thus

The powers of x are $1 = x^0$, $x = x^1$, x^2 , x^3 , \dots

A fraction looks like $\frac{1}{2}$

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To get Greek letters in math mode you just use their Endlish names precided by a backslash: α , β , γ etc.

To go a displayed line of mathematics you put the formula between either double dollar signs, \dots or between $[\dots]$. As an example

The area under the graph $y=f(x)$ is given by the integral

$\int_a^b f(x)dx$.

∂D

If D is a domain, then its boundary is ∂D and the integral over the boundary is

$\int_{\partial D}$

$\int_{\partial D} f(z)dz$.

$u(x,y)$

If $u(x,y)$ depends on the variables x and y then the

partial derivative with respect to x is

$$\frac{\partial u}{\partial x}(x, y) = u_x(x, y) = \lim_{h \rightarrow 0} \frac{u(x+h, y) - u(x, y)}{h}.$$

\$\$

To make x bold in math mode use the command `\verb+\mathbf{x}+`. Thus

\$\$

$$\mathbf{x}_u = \frac{\partial \mathbf{x}}{\partial u}$$

\$\$

and

\$\$

$$\mathbf{x}_{uv} = \nabla_{\mathbf{x}} \mathbf{x}_u \mathbf{x}_v + II(\mathbf{x}_u, \mathbf{x}_v) \mathbf{n}$$

\$\$

gives

The area under the graph $y = f(x)$ is given by the integral

$$\int_a^b f(x) dx.$$

If D is a domain, then its boundary is ∂D and the integral over the boundary is

$$\int_{\partial D} f(z) dz.$$

If $u = u(x, y)$ depends on the variables x and y then the partial derivative with respect to x is

$$\frac{\partial u}{\partial x}(x, y) = u_x(x, y) = \lim_{h \rightarrow 0} \frac{u(x+h, y) - u(x, y)}{h}.$$

To make x bold in math mode use the command `\mathbf{x}`. Thus

$$\mathbf{x}_u = \frac{\partial \mathbf{x}}{\partial u}$$

and

$$\mathbf{x}_{uv} = \nabla_{\mathbf{x}} \mathbf{x}_v + II(\mathbf{x}_u, \mathbf{x}_v) \mathbf{n}$$

Your assignment: Use LaTeX to typeset the following and send me a pdf file of the output. If you get stuck and need help you can e-mail the LaTeX file and I can do my best to debug it. *Hints:* The plus or minus sign \pm is `\pm`. The double integral \iint is `\iint`. The calligraphy form of a capital letter in math mode, say \mathcal{A} , \mathcal{B} , and \mathcal{C} is `\mathcal{A}`, `\mathcal{B}`, and `\mathcal{C}`. The command to get \dot{u} is `\dot{u}` and

to get \ddot{u} is `\ddot{u}`. To get a capital γ , that is Γ , the command is `\Gamma`.

1. If γ is a simple closed curve in the plane then its **total curvature** is

$$\int_{\gamma} \kappa(s) ds = \pm 2\pi.$$

2. If T is a geodesic triangle in a surface, M , with angles α , β , and γ and K is the *Gauss Curvature*, then

$$\alpha + \beta + \gamma - \pi = \iint_T K dA$$

3. If $\mathbf{c}: [a, b] \rightarrow M$ is a curve in the surface M , then it is a **geodesic** of M if and only if $\mathbf{c}''(s)$ is always perpendicular to M .

4. If $u(t)$ minimizes the functional

$$\mathcal{F}[y] = \int_a^b L(t, y(t), \dot{y}(t)) dt$$

with given endpoint values, then u satisfies the **Euler-Lagrange equation**

$$L_y(t, u(t), \dot{u}(t)) - \frac{d}{dt} L_{\dot{y}}(t, u(t), \dot{u}(t)) = 0.$$

5. A formula that will be useful to use in the future is

$$\mathbf{f}_{u^i u^j} = \sum_k \Gamma_{ij}^k \mathbf{f}_{u^k} + L_{ij} \mathbf{n}.$$