Mathematics 551 Homework, April 8, 2020

First here is the basic template you will for this homework:

```
\documentclass[12pt]{amsart} % Use the AMS article class 12 point type size


\begin{document}  % You put your input between these two

\end{document}
```

If you type:

```
Here are the basics of using LaTeX to typeset  text (with no
mathematics).  You just type in the text and LaTeX will take
care of the spacing, breaks and the like.

To start a new paragraph just skip one or more lines.
Your can \emph{emphasize
one or more words} or make them \textbf{bold}.
It is even possible to
\underline{underline stuff}.
```

You will get:

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Typing

```
When type setting mathematics in a paragraph you change to \emph{math
mode}.  This is done putting the formula, either between
two dollar signs (that is \$ \dots \$) or between
\verb+\(+ and \verb+\)+.  Thus

The powers of $x$ are $1=x^0$, $x=x^1$, $x^2$, $x^3$, \ldots

A fraction looks like $\frac{1}{2}$
```

```
To the best of my knowledge Issac Newton was the first
person to use fractional exponents to denote roots, that is
the first to write such statements as \(\sqrt{a} =
a^\frac{1}{2}\) and \( \sqrt[3]{b} = b^\frac{1}{3}\)

The notation $\int_a^b f(x)dx$ for the integral is is due to
Leibniz. The sigma notation for a sum $\sum_{n=5}^{100} a_n$
is due to Euler.

I personally prefer the \$ \dots \$ version, but you use
what you prefer.

To get greek letters in math mode you just use their Endlish names
precided by a backslash:  $\alpha$, $\beta$, $\gamma$ etc.
```

gives

When type setting mathematics in a paragraph you change to *math mode*. This is done putting the formula, either between two dollar signs (that is $ ... $) or between `\(` and `\)`. Thus

The powers of x are $1 = x^0$, $x = x^1$, x^2, x^3, ...

A fraction looks like $\frac{1}{2}$

To the best of my knowledge Issac Newton was the first person to use fractional exponents to denote roots, that is the first to write such statements as $\sqrt{a} = a^\frac{1}{2}$ and $\sqrt[3]{b} = b^\frac{1}{3}$

The notation $\int_a^b f(x)dx$ for the integral is is due to Leibniz. The sigma notation for a sum $\sum_{n=5}^{100} a_n$ is due to Euler.

I personally prefer the $ ... $ version, but you use what you prefer.

To get Greek letters in math mode you just use their Endlish names precided by a backslash: α, β, γ etc.

To go a displayed line of mathematics you put the formula between either double dollar signs, $$ ... $$ or between `\[` ... `\]`. As an example

```
The area under the graph $y=f(x)$ is given by the integral
$$
\int_a^b f(x)dx.
$$
If $D$ is a domain, then its boundary is $\partial D$
and the integral over the boundary is
\[
\int_{\partial D} f(z)dz.
\]
If $u=u(x,y)$ depends on the variables $x$ and $y$ then the
```

```
partial derivative with respect to $x$ is
$$
\frac{\partial u}{\partial x}(x,y) = u_x(x,y) =
  \lim_{h \to 0} \frac{u(x+h,y) - u(x,y)}{h}.
$$
To make $x$ bold in math mode  use
the command \verb+\mathbf{x}+.  Thus
$$
\mathbf{x}_u = \frac{\partial \mathbf{x}}{\partial u}
$$
and
$$
\mathbf{x}_{uv} = \nabla_{\mathbf{x}_u}\mathbf{x}_v +
II(\mathbf{x}_u,\mathbf{x}_v)\mathbf{n}
$$
```

gives

The area under the graph $y = f(x)$ is given by the integral

$$\int_a^b f(x)dx.$$

If D is a domain, then its boundary is ∂D and the integral over the boundary is

$$\int_{\partial D} f(z)dz.$$

If $u = u(x, y)$ depends on the variables x and y then the partial derivative with respect to x is

$$\frac{\partial u}{\partial x}(x,y) = u_x(x,y) = \lim_{h \to 0} \frac{u(x+h,y) - u(x,y)}{h}.$$

To make x bold in math mode use the command `\mathbf{x}`. Thus

$$\mathbf{x}_u = \frac{\partial \mathbf{x}}{\partial u}$$

and

$$\mathbf{x}_{uv} = \nabla_{\mathbf{x}_u}\mathbf{x}_v + II(\mathbf{x}_u,\mathbf{x}_v)\mathbf{n}$$

Your assignment: Use LaTeX to typeset the following and send me a pdf file of the output. If you get stuck and need help you can e-mail the LaTeX file and I can do my best to debug it. *Hints:* To say that z is in the set D, that is $z \in D$, type `z\in D`. The double integral $\iint$ is `\iint`. The infinity symbol ∞ is `\infty`.

The calligraphy form of a capital letter in math mode, say $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ is `\mathcal{A}`, `\mathcal{B}`, and `\mathcal{C}` The command to get $\dot{u}$ is `\dot{u}` and to get $\ddot{u}$ is `\ddot{u}`. To get a capital γ, that is Γ, the command is `\Gamma`.

1. If D is a bounded domain with nice boundary and $f(z)$ is analytic on the closure of D then the **Cauchy Integral Theorem** tells us that

$$\int_{\partial D} f(z)dz = 0.$$

2. With D as in Problem 1, if $z \in D$, then the *Cauchy integral formula* for $f(z)$ is

$$f(z) = \frac{1}{2\pi i} \int_{\partial D} \frac{f(w)}{w - z} dw$$

and the formula for the derivative is

$$f'(z) = \frac{1}{2\pi i} \int_{\partial D} \frac{f(w)}{(w - z)^2} dw.$$

3. Our proof of the Cauchy Integral Theorem was based on the *Cauchy-Riemann equations*

$$u_x = v_y$$

and

$$u_y = -v_x$$

and *Green's Theorem*

$$\int_{\partial D} Pdx + Qdy = \iint_D (-P_y + Q_x)dxdy.$$

4. *Laurent's Theorem* tells us that an near an isolated singularity, a, of the analytic function $f(z)$ that there is a series expansion

$$f(z) = \sum_{n=-\infty}^{\infty} c_n (z - a)^n$$

and that the coefficients, c_n, are given by

$$c_n = \frac{1}{2\pi i} \int_{|z-a|=\rho} \frac{f(w)}{(z - a)^{n+1}} dz.$$

The number c_{-1} is the **Resdue** of $f(z)$ and $z = a$.

5. Let D be a bounded domain with a nice boundary and $f(z)$ be a function that is analytic on the closure of D expect at isolated singularities a_1, a_2, $\ldots$, a_m inside of D. Let R_j be the residue of $f(z)$ and $z = a_j$. Then the **Residue Theorem** is

$$\int_{\partial D} f(z)dz = 2\pi i(R_1 + R_2 + \cdots + R_m).$$

That is the integral of $f(z)$ over the boundary of D is $2\pi i$ times the sum of the residues of $f(z)$ inside of D.