Mathematics 552 Homework, February 20, 2020

Problem 1. (a) Draw the region, D, defined by $1 < |z| < 2$ and $0 < \operatorname{Arg}(z) < \pi/2$.

(b) What is the image of D under the map $f(z) = z^2$. That is what is the set of points $\{z^2 : z \in D\}$. Draw a picture of the image.*Hint:* This is one of the many cases where writing $z = re^{i\theta}$ in polar form makes things easier.

(c) What is the image of D under the map $g(z) = z^3$? Draw the picture.

(d) What is the image of D under the map $h(z) = 1/z$. Draw the picture of the image. □

Problem 2. Let $f = u + iv$ be analytic in the open set U. The gradients of u and v are defined as in vector calculus as

$$\nabla u = (u_x, u_y), \qquad \nabla v = (v_x, v_y).$$

Use the Cauchy-Riemann Equations to show that ∇u and ∇ are perpendicular. That is show that their dot product is zero. What does this say about the curves defined by $u = a$ and $v = b$ where u and v are constants?

Problem 3. Here is some practice in computing line integrals.

(a) This problem is very like the solved problem 4.1 in the text. Compute the line integral

$$\int_{(0,0)}^{(1,2)} (x + y^2)\, dx + (x - 2y)\, dy$$

(i) Along the straight line segment from $(0, 0)$ to $(1, 2)$.

(ii) Along the parabola $x = t$, $y = 2t^2$ with $0 \le t \le 1$.

(b) Compute the line integral

$$\int_\gamma z^2\, dz$$

(i) Where γ is the curve $z(t) = t^2 + t^3\, i$ for $-1 \le t \le 1$.

(ii) Where γ is the circle $|z| = 1$ traversed in the positive (that is counterclockwise) direction.