Mathematics 300

Quiz 10 Name: Key

You must show your work to get full credit.

1. Define the following: (a) The integer n is ***even***.

There is an integer q so that $n = 2q$.

(b) The integer n is ***odd***.

There is an integer q so that $n = 2q+1$

(c) The integer a ***divides*** the integer b (in symbols $a \mid b$).

There is an integer q so that $b = qa$.

(d) For integers a, b, n the a is ***congruent to*** b ***modulo*** n. (in symbols $a \equiv b \pmod{n}$.)

$n \mid (b-a)$

2. Prove or give a counterexample:
(a) The integer 0 is even.

True $0 = 2 \cdot 0$, that is $0 = 2q$ where $q = 0$.

(b) The integer 42 is even.

True $42 = 2q$ where $q = 21$

(c) If n is even, then $n+1$ is odd.

True Assume n is even Then $n = 2q$ for some integer q. Thus

$n+1 = 2q+1$ so $n+1$ is odd.

(d) If n is even, then $n-6$ is even.

True Assume n is even. Then $n = 2q$ for some integer q. Then

$n+6 = 2q+6$

$= 2(q+3) = 2q'$

where $q' = q+3$ is an integer by closure properties. Thus $n+6$ is even.

3. (a) Define what it means for a, b, and c to be a Pythogonian triple.

a, b, and c are positive integers with
$a^2+b^2=c^2$

(b) Find all Pythogonian triples of the form m, $m+1$, and $m+2$.

If $a=m$, $b=m+1$, $c=m+2$ is a Pythogonian triple

$$a^2+b^2=c^2$$

i.e.

$$m^2+(m+1)^2=(m+2)^2$$

$$m^2+m^2+2m+1=m^2+4m+4$$

$$m^2-2m-3=0$$

$$(m-3)(m+1)=0$$

$$m=3,-1$$

only positive solution count, so $m=3$ and so the only triple is
3, 4, 5

4. Make a truth table for $P \wedge Q \to P \vee Q$.

P	Q	$P \wedge Q$	$P \vee Q$	$P \wedge Q \to P \vee Q$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

5. Use truth tables to explain why $P \to Q$ and $\neg Q \to \neg P$ are logially equivalent.

Table for $P \to Q$

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

Table for $\neg Q \to \neg P$

P	Q	$\neg Q$	$\neg P$	$\neg Q \to \neg P$
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

They have the same truth tables so they are logically equivalent.

6. For the statement $P \to Q$

(a) What is the converse? $Q \to P$

(b) What is the negatrion? $P \wedge \neg Q$

(c) What is the contrapositive? $\neg Q \to \neg P$

7. For the statement: If the weather is good, I will go hiking.

(a) What is the converse.

If I go hiking, the weather is good

(b) What is the negatation. The weather is good, and I do not go hiking

(c) What is the contrapositive. If I do not go hiking, the weather is not good.

8. (a) Write $\{x \in \mathbb{N} : (x-2)(x+5)(3x-7) = 0\}$ in roster notation.

$(x-2)(x+5)(3x-7)=0$ has solutions $2, -5, \frac{7}{3}$. On 2 is a natural number

$\{2\}$

(b) Write $\{x \in \mathbb{Z} : (x-2)(x+5)(3x-7) = 0\}$ in roster notation.

$2, -5 \in \mathbb{Z}$, $\frac{7}{3} \notin \mathbb{Z}$ so $\{2, -5\}$

(c) Write $\{x \in \mathbb{R} : (x-2)(x+5)(3x-7) = 0\}$ in roster notation.

$\{2, -5, \frac{7}{3}\}$

9. (a) What is the negation of the statement: Every Math 300 student is a mathematics major.

Some Math 300 major is not a mathematics major.

(b) What is the negation of the statement: Some Math 300 student is a music major.

No Math 300 student is a music major.

10. Prove: if a divides x and b divides y, then ab divides $x^2y + xy^2$.

Assume $a \mid x$ and $b \mid y$. Then there are integers p, q with $x = pa$, $y = qb$. So

$x^2y + xy^2 = (pa)^2(qb) + (pa)(qb)^2$

$= ab(ap^2q + bpq^2)$

$= abm$

where $m = ap^2q + bpq^2$ by closure properties. Thus $ab \mid x^2y + xy^2$

11. Prove or given a ~~counder example~~ counterexample: If n is odd, then $n^2 + 2 \equiv 3 \pmod 4$.

True. Assume n is odd. Then $n = 2q+1$ for some integer q. Then

$n^2+2 = (2q+1)^2+2$

$= 4q^2+4q+1+2$

$= 4(q^2+q)+3$

$= 4p+3$

where $p = q^2+q$ is an integer by closure properties.

Thus $(n^2+2) - 3 = 4p$

Thus $4 \mid ((n^2+2)-3)$

so $n^2+2 \equiv 3 \pmod 4$

12. Prove: If $a \equiv b \pmod m$ and $x \equiv y \pmod m$, then $a + x \equiv b + y \pmod m$.

Assume $a \equiv b \pmod m$ and $x \equiv y \pmod m$.

Then $m \mid (a-b)$ and $m \mid (x-y)$

so there are integers p, q with

$a - b = mp$ $\quad x - y = mq$.

so $a = b + mp$ $\quad x = y + mq$.

Then $(a+x) - (b+y) = (b+mp + y+mq) - b+y$

$= m(p+q)$

$= mk$

where $k = p+q$ is an integer by closure properties. Thus $m \mid (a+x) - (b+y)$ and therefore $a+x \equiv b+y \pmod m$