Mathematics 300

Quiz 13 Name: Key

You must show your work to get full credit.

Proposition. *For all real numbers a and b the inequality*

$$2ab \le a^2 + b^2$$

holds.

Proof. By algebra

$$2ab \le a^2 + b^2$$

$$0 \le a^2 - 2ab + b^b \qquad \text{(subtract } 2ab \text{ from both sides)}$$

$$0 \le (a - b)^2 \qquad \text{(factor)}$$

and this last step is true because the square any real numbers is ≥ 0. □

1. This proof is poorly written. Rewrite to make it better. *Hint:* Note that by starting with $2ab \le a^2 + b^2$ the proof is pretty much starting with what it is trying to prove.

Since the square of any real number is ≥ 0 we have

$$0 \le (a-b)^2$$

$$0 \le a^2 - 2ab + b^2 \qquad \text{(expanding)}$$

$$2ab \le a^2 + b^2 \qquad \text{add } 2ab \text{ to both sides}$$

done.

2. Prove: If $a^3 + a$ is irrational, then a is irrational.

We prove the contrapositive:

If a is rational, then a^3+a is ration.

Assume a is rational. Then $a = \frac{p}{q}$ for integers p and q with $q \neq 0$.

Then

$$a^3 + a = \left(\frac{p}{q}\right)^3 + \left(\frac{p}{q}\right)$$

$$= \frac{p^3}{q^3} + \frac{p}{q}$$

$$= \frac{p^3 + pq^2}{q^3}$$

$$= \frac{p'}{q'}$$

where $p' = p^3 + pq^2$ and $q' = q^3$ are integers and $q' \neq 0$. So a^3+a is rational

done