

You must show your work to get full credit.

1. Add the following fractions $\frac{y+x}{x} + \frac{x}{y}$.

The sum is

$$\frac{y^2 + xy + x^2}{xy}$$

$$\frac{y+x}{x} + \frac{x}{y} = \frac{(y+x)y}{xy} + \frac{x^2}{xy} = \frac{y^2 + xy + x^2}{xy}$$

2. Prove: The integer n is odd if and only if n^2 is odd.

We need to prove two implications

(a) If n is odd, then n^2 is odd.

Assume n is odd. Then $n = 2q + 1$ for some integer q . Then

$$\begin{aligned} n^2 &= (2q+1)^2 \\ &= 4q^2 + 4q + 1 \\ &= 2(2q^2 + 2q) + 1 \\ &= 2h \end{aligned}$$

where $h = 2q^2 + 2q$ is an integer. Thus n^2 is odd.

(b) If n^2 is odd, then n is odd.

We prove the contrapositive:

If n is even, then n^2 is even

Assume n is even. Then $n = 2q$ for some integer q . Thus

$$\begin{aligned} n^2 &= (2q)^2 \\ &= 2(2q^2) \\ &= 2h \end{aligned}$$

where $h = 2q^2$ is an integer. Thus n^2 is even. done