

Mathematics 300

Quiz 15

Name: Key

You must show your work to get full credit.

1. Expand the following $(x-1)^4$.

$$\begin{array}{ccccccc} & & & & 1 & & \\ & & & & 1 & & \\ & & 1 & & 2 & & 1 \\ & 1 & & 3 & & 3 & \\ 1 & & 4 & & 6 & & 4 & \\ & & & & & & & 1 \end{array}$$

$$(x-1)^4 = \underline{x^4 - 4x^3 + 6x^2 - 4x + 1}$$

$$\begin{aligned} (x-1)^4 &= x^4 + 4x^3(-1) + 6x^2(-1)^2 + 4x(-1)^3 + (-1)^4 \\ &= x^4 - 4x^3 + 6x^2 - 4x + 1 \end{aligned}$$

2. We know that $\sqrt{2}$ is irrational. Use this fact and a proof by contradiction to show that $3\sqrt{2} - 1$ is irrational.

Toward a contradiction assume $3\sqrt{2} - 1$ is rational.

Then $3\sqrt{2} - 1 = \frac{a}{b}$ where a, b are integers

and $b \neq 0$. We solve for $\sqrt{2}$

$$3\sqrt{2} - 1 = \frac{a}{b}$$

$$3\sqrt{2} = \frac{a}{b} + 1 = \frac{a}{b} + \frac{b}{b} = \frac{a+b}{b}$$

$$\sqrt{2} = \frac{a+b}{3b} = \frac{p}{q}$$

where $p = a+b$ and $q = 3b$ are integers. This

implies $\sqrt{2}$ is rational, contradicting that

we know $\sqrt{2}$ is irrational. done