Mathematics 300

Quiz 19 Name: Key

You must show your work to get full credit.

1. Prove: For all integers n the number $n^2 + 3n$ is even.

Proof The integer n is either even or odd.

Case 1 n is even. Then $n = 2q$ where q is an integer. So

$$\begin{aligned} n^2+3n &= (2q)^2 + 3(2q) \\ &= 4q^2+6q \\ &= 2(2q^2+3) \\ &= 2m \end{aligned}$$

where $m = 2q^2+3$ is an integer. Thus n^2+3n is even

Case 2 n is odd. Then $n = 2q+1$ for some integer q and

$$\begin{aligned} n^2+3n &= (2q+1)^2 + 3(2q+1) \\ &= 4q^2+4q+1+6q+3 \\ &= 4q^2+10q+4 \\ &= 2(2q^2+5q+2) \\ &= 2m \end{aligned}$$

where $m = 2q^2+5q+2$ is an integer. Thus n^2+3n is even.

2. Find all solutions to $|2x + 3| = 9$. The solutions are: $3, -6$

Case 1 $2x+3=9$,
$2x = 6$
$x = 3$

Case 2 $2x+3 = -9$
$2x = -12$
$x = -6$

Check:
$x = 3$
$|2x+3| = |6+3| = 9$ ✓
$x = -6$
$|2x+3| = |-12+3| = |-9| = 9$ ✓