Mathematics 300

Quiz 20 Name: Key

You must show your work to get full credit.

Division Algorithm. *Let a and b be integers with b positive. Then there are unique integers q and r such that*

$$a = qb + r \qquad 0 \leq r < b.$$

$\square$

1. Use the divisor algorithm to and proof by cases to show For any integer n the number $n(n-1)(n+1)$ is divisible by 3.

Write $n = 3q + r$ where $r = 0, 1, 2$.

<u>case 1</u> $r = 0$, then $n = 3q$ so

$$n(n-1)(n+1) = 3q(3q-1)(3q+1) = 3m$$

where $m = q(3q-1)(3q+1)$ is an integer. So 3 divides $n(n-1)(n+1)$ in this case.

<u>case 2</u> $r = 1$, then $n = 3q+1$ and

$$n(n-1)(n+1) = (3q+1)(3q)(3q+2) = 3(3q+1)q(3q+2) = 3m$$

where $m = (3q+1)q(3q+2)$ is an integer and thus 3 divides $n(n-1)(n+1)$ in this case

<u>case 3</u> $r = 2$, then $n = 3q+2$ and

$$n(n-1)(n+1) = (3q+2)(3q+1)(3q+3) = 3(3q+2)(3q+1)(q+1) = 3m$$

where $m = (3q+2)(3q+1)(q+1)$ is an integer. Thus 3 divides $n(n-1)(n+1)$ in this case also.

<u>done</u>