Mathematics 300

Quiz 25 Name: Key

You must show your work to get full credit.

1. Let $y = xe^x$.
(a) Compute the first three derivatives of y

$y' =$ ________ $y'' =$ ________ $y''' =$ ________

$y = x e^x \quad y' = x' e^x + x(e^x)' = e^x + x e^x = (x+1)e^x$

$y'' = (x+1)' e^x + (x+1)(e^x)' = e^x + (x+1)e^x = (x+2)e^x$

$y''' = (x+2)' e^x + (x+2)(e^x)' = e^x + (x+2)e^x = (x+3)e^x$

(b) Make a conjecture at for a formula for the n-th derivative of y. $y^{(n)} =$ $(x+n)e^x$

(c) Prove your conjecture using induction.
Base case: $n=1$ then $y' = (x+1)e^x$ which is $y^{(n)} = (x+n)e^x$ when $n=1$

Induction step:

Induction hypothesis: The kth derivative of y is

$y^{(k)} = (x+k)e^x$

Take the derivative of this to get

$y^{(k+1)} = (y^{(k)})' = ((x+k)e^x)'$

$= (x+k)' e^x + (x+k)(e^x)'$

$= 1 e^x + (x+k)e^x$

$= (x+(k+1))e^x$

which is $y^{(n)} = (x+n)e^x$ with $n = k+1$.
This completes the induction.

2. The contrapositive of $P \to Q$ is $\neg Q \to \neg P$. Use truth tables to show these are logically equivalent.

P	Q	$P \to Q$	$\neg P$	$\neg Q$	$\neg Q \to \neg P$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

The truth tables for $P \to Q$ and $\neg Q \to \neg P$ are the same so they are logically equivalent.