

Mathematics 300 Homework, March 26, 2022.

Problem 1. Prove the following using mathematical induction.

(a) $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$

(b) $1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}.$

(c) $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

(d) $1 + 2 + 2^2 + 2^3 + \cdots + 2^n = 2^{n+1} - 1.$

Problem 2. Let a be a constant and let

$$y = xe^{ax}.$$

Prove the n derivative of y is

$$\frac{d^n y}{dx^n} = (a^n + na^{n-1})e^{ax}.$$

Problem 3. This problem lets you review a bit of calculus.

(a) Let n be a positive integer. Use integration by parts to show

$$\int_0^\infty x^n e^{-x} dx = n \int_0^\infty x^{n-1} e^{-x} dx.$$

(b) Use Part (a) and induction to so that for all positive integers n the equality

$$\int_0^\infty x^n e^{-x} dx = n!$$

holds.