Mathematics 300 Homework, April 6, 2022.

Problem 1. Be able to give a precise statement of the ***division algorithm***: If a and b are integers with $b > 0$, then there are unique integers q and r such that

$$a = qb + r \qquad \text{and} \qquad 0 \leq r < b.$$

□

Here is an example of similar to done in class.

Proposition 1. *Let c_n be defined recursively by*

$$c_1 = 5 \qquad c_{n+1} = \frac{2}{3}c_n + 10.$$

Then $c_n < 30$ for all n.

Proof. We use induction on n.

Base case: $n = 1$. $c_1 = 5 < 30$ holds.

Induction step: Our induction hypothesis is $c_k < 30$.

We will use the induction hypothesis to prove $c_{k+1} < 30$.

$$\begin{aligned} c_{k+1} &= \frac{2}{3}c_k + 10 \\ &< \frac{2}{3}(30) + 10 \qquad \text{(This is where } c_k < 30 \text{ is used)} \\ &= 30. \end{aligned}$$

This completes the induction. □

Problem 2. If $a_1, a_2, \ldots$ is defined by

$$a_1 = 10, \qquad a_{n+1} = \frac{1}{3}a_n + 42.$$

Show $a_n < 63$ for all n. □

Problem 3. Do Problem 17 on page 198 of the text.

Problem 4. On pages 208–2-9 of the text do Problems 10, 11, 13.