Mathematics 300 Homework, January 31, 2022.

For the quiz on Wednesday you will have to know:

Definition 1. Let a, b, n be integers then

$$a \equiv b \mod n$$

if and only if $n \mid (a - b)$

Problem 1. Show a is odd if and only if $a \equiv 1 \mod 2$.

Proof. Since this an "if and only if" statement we need to prove two implication.

First we show a is odd implies $a \equiv 1 \mod 2$. Assume a is odd. Then

$$a = 2q + 1$$

for some integers q. By algebra this implies

$$a - 1 = 2q.$$

This is the definition of $2 \mid (a - 1)$ which in turn is the defintion of $a \equiv 1 \mod 2$. Thus a odd implies $a \equiv 1 \mod 2$.

We now show that $a \equiv 1 \mod 2$ implies a is odd. Assume that $a \equiv 1 \mod 2$. Then $2 \mid (a - 1)$ which implies

$$a - 1 = 2q$$

for some integers q. By algebra this gives

$$a = 2q + 1$$

which is our definition of a being odd. This $a \equiv 1 \mod 2$ implies a odd. □

On Page 97 of the text do Problems 7 (see your class notes for this), 8, 9, 10. As the some problem on the quiz will like one of these, so you should either make sure you know how to do them, or ask a question about it in class.