Mathematics 300

Quiz 10 **Name:**_____

You must show your work to get full credit.

1. Define the following: (a) The integer n is ***even***.

(b) The integer n is ***odd***.

(c) The integer a ***divides*** the integer b (in symbols $a \mid b$).

(d) For integers a, b, n the a is ***congruent to*** b ***modulo*** n. (in symbols $a \equiv b \pmod{n}$.)

2. Prove or give a counterexample:
(a) The integer 0 is even.

(b) The integer 42 is even.

(c) If n is even, then $n + 1$ is odd.

(d) If n is even, then $n - 6$ is even.

3. (a) Define what it means for a, b, and c to be a Pythogonian triple.

(b) Find all Pythogonian triples of the form m, $m+1$, and $m+2$.

4. Make a truth table for $P \wedge Q \to P \vee Q$.

5. Use truth tables to explain why $P \to Q$ and $\neg Q \to \neg P$ are logially equivalent.

6. For the statement $P \to Q$

(a) What is the converse?

(b) What is the negatrion?

(c) What is the contrapositive?

7. For the statement: If the weather is good, I will go hiking.

(a) What is the converse.

(b) What is the negatation.

(c) What is the contrapositive.

8. (a) Write $\{x \in \mathbb{N} \ : (x-2)(x+5)(3x-7) = 0\}$ in roster notation.

(b) Write $\{x \in \mathbb{Z} \ : (x-2)(x+5)(3x-7) = 0\}$ in roster notation.

(c) Write $\{x \in \mathbb{R} \ : (x-2)(x+5)(3x-7) = 0\}$ in roster notation.

9. (a) What is the negation of the statement: Every Math 300 student is a mathematics major.

(b) What is the negation of the statement: Some Math 300 student is a music major.

10. Prove: if a divides x and b divides y, then ab divides $x^2y + xy^2$.

11. Prove or given a counterexample: If n is odd, then $n^2 + 2 \equiv 3 \pmod 4$.

12. Prove: If $a \equiv b \pmod m$ and $x \equiv y \pmod m$, then $a + x \equiv b + y \pmod m$.