Mathematics 300

Quiz 13 **Name:**_____

You must show your work to get full credit.

Proposition. *For all real numbers a and b the inequality*

$$2ab \leq a^2 + b^2$$

holds.

Proof. By algebra

$$\begin{aligned} 2ab &\leq a^2 + b^2 \\ 0 &\leq a^2 - 2ab + b^2 && \text{(subtract } 2ab \text{ from both sides)} \\ 0 &\leq (a-b)^2 && \text{(factor)} \end{aligned}$$

and this last step is true because the square any real numbers is ≥ 0. □

1. This proof is poorly written. Rewrite to make it better. *Hint:* Note that by starting with $2ab \leq a^2 + b^2$ the proof is pretty much starting with what it is trying to prove.

2. Prove: If $a^3 + a$ is irrational, then a is irrational.