Mathematics 300

Quiz 23 **Name:**_____

You must show your work to get full credit.

1. Define the following:
(a) $a \equiv b \pmod{n}$.

(b) a is ***divisor*** of b.

(c) r is a ***rational*** number.

(d) r is ***irrational*** number.

(e) The ***absolute value*** of the number a.

2. State the following:
(a) The ***division algorithm***.

(b) The contrapositive of the statement: If n^3 is divisible by 3, then n is divisible by 3.

3. Prove or give a counterexample: $(x+y)^5 \equiv x^5 + y^5 \pmod{5}$.

4. Compute the quotient and remainder when 7 is divided into -19.

$q =$ _____ $r =$ _____

5. Prove: For any real numbers a and x, if $|x - 1| = a$, then $x = a + 1$ or $x = -a + 1$.

6. Prove or give a counter example: If α is irrational, then $\dfrac{\alpha - 1}{\alpha + 1}$ is irrational.

7. Prove: For all integers n the number $n^3 + 5n$ is divisible by 3.

8. We have shown that $\sqrt{3}$ is irrational. Use this to show that for any real number x that either x or $x - \sqrt{3}$ is irrational.

9. Prove or give a counterexample: If $3a \equiv 0 \pmod 6$, then $a \equiv 0 \pmod 6$.

10. Prove or give a counterexample: If $3a \equiv 0 \pmod{5}$, then $a \equiv 0 \pmod{5}$.

11. Prove that if a and b have the same remainder when divided by n, then $a \equiv b \pmod{n}$.