Mathematics 300

Quiz 31 **Name:**__________________________________

You must show your work to get full credit.

1. Use induction on a to prove the following special case of the ***division algorithm***. If a and b are positive integers, there there are integers q and r with

$$a = qb + r \qquad \text{with} \qquad 0 \leq r < b.$$

(Note we are not proving the uniqueness of q and r.)

2. Let $A = \{1, 2, 3, 4, 5, 6\}$ and B be the set of even integers between -3 and 9.

Write B as the list of its elements. ________________________

What is $A \cup B$? ________________________

What is $A \cap B$? ________________________

What is $A - B$ ________________________

3. Draw the Venn diagrams for

(a) $A \cap B^c$

(b) $(A - B) \cup (B - A)$

(c) $(A \cup B) - (A \cap B)$.

(d) Are $(A - B) \cup (B - A)$ and $(A \cup B) - (A \cap B)$ equal? Why?

4. Let a_n be defined recursively by

$$a_{n+1} = \frac{1}{2}a_n + 100, \qquad a_1 = 2$$

Prove that $a_n \leq 300$ for all n.

5. Define the ***Fibonacci numbers*** by the recursion

$$f_{n+2} = f_{n+1} + f_n, \qquad f_1 = 1, \quad f_2 = 1$$

(a) Compute

$f_3 =$ __________ $f_4 =$ __________ $f_5 =$ __________ $f_6 =$ __________ $f_7 =$ __________

(b) Prove

$$\sum_{k=1}^{n} f_k = f_{k+2} - 1.$$

6. Give an example of two sets with $A \cap B \neq A \cup B$. The example should be explicitly given sets, not just a Venn diagram.

7. Use Venn diagrams to show for sets A, C and B that $(A \cup C) \cap B = (A \cap B) \cup (C \cap B)$

8. Let $A_1, A_2, \ldots, A_n, B$ be subsets of a set U. Use induction to show

$$(A_1 \cup A_2 \cup \cdots \cup A_n) \cap B = (A_1 \cap B) \cup (A_2 \cap B) \cup \cdots \cup (A_n \cap B)$$