Mathematics 300

Quiz 32 **Name:**________________________________

You must show your work to get full credit.

1. Prove or give a counter example: Every even integer is the some of two odd integers.

2. Use Venn diagrams to show that $(A \cup B) - C = (A - C) \cup (B - C)$.

3. What is the quotient and reminder when -32 is divided by 7?

Quotient is ____________________ Remainder is ____________________

4. Find the sum $S = \sum_{k=0}^{10} 3(2)^k$.

5. Find the sum of the following twenty terms $31 + 32 + \cdots + 50$.

The sums is ____________________

6. What is wrong with the following statement of the division algorithm. "For unique integers a and b with $b > 0$ there are integers q and r with

$$a = qb + r \qquad \text{and} \qquad 0 \leq r < b.$$

7. If chicken nuggets are only sold in boxes with two or three nuggets, show that it is possible to buy exactly n nuggets for any $n \geq 2$.

8. Use induction to show that $10^n \equiv 1 \pmod{9}$ for all integers $n \geq 1$.

9. Let $f(x) = (x+1)e^x$.

(a) Compute the first four derivatives of $f(x)$.

$f'(x) =$ ________________ $f''(x) =$ ________________

$f'''(x) =$ ________________ $f^{(4)}(x) =$ ________________

(b) Make a conjecture for a formula for the n-th derivative $f^{(n)}(x)$.

$f^{(n)}(x) =$ ________________

(c) Prove your conjecture.

10. Let a_n be defined by

$$a_{n+1} = 3a_n - 8, \qquad a_0 = 6$$

(a) Find the following

$a_1 =$ __________ $a_2 =$ __________ $a_3 =$ __________

(b) Prove $a_n = 2(3^n) + 4$

11. (a) Define $a \equiv b \pmod{n}$.

(b) Prove that if a and b have the same remainder when divided by n that $a \equiv b \pmod{n}$.