Mathematics 300

Quiz 35 **Name:**_____

You must show your work to get full credit.

1. Let $A = \{x \in \mathbb{Z} : x \equiv 2 \pmod 5\}$ and $B = \{y \in \mathbb{Z} : y \equiv 7 \pmod{10}\}$.

(a) List at least 5 elements of A. _____

(b) List at least 5 elements of B. _____

(c) Is $A \subseteq B$, prove or give a counterexample.

(d) Is $B \subseteq A$, prove or give a counterexample.

2. Draw a Venn diagram with sets A, B, and C with $A \subseteq C$, $B \subseteq C$ and with A and B disjoint (that is $A \cap B = \emptyset$).

3. Prove the sets $S = \{x \in \mathbb{Z} : x \equiv 2 \pmod 7\}$ and $T = \{y \equiv 3 \pmod{14}\}$ are disjoint.

4. Prove: If k is an integer and $3 \mid k^2$, then $3 \mid k$.

5. Use Problem 4 to prove $\sqrt{3}$ is irrational.

6. For integers x and y prove: If $3 \mid x$ and $2 \mid y$, then $12 \mid (5xy^2 + 10xy)$.

7. What is the negation of the statement: "For all $x \in \mathbb{R}$ there is a $n \in \mathbb{N}$ with $n > x$".

8. (a) Define what it means for r to be a rational number.

(b) Prove or give a counterexample: If a and b are irrational, then so is the product ab.

(c) Prove or give a counterexample: If $a \neq 0$ is rational and b is irrational, then the product ab.