Mathematics 300

Quiz 36 **Name:**_____

You must show your work to get full credit.

1. Prove if a and b are odd integers, then $(a-1)(b-1)$ is divisible by 4.

Recall at ***Pythagorean triple*** is a list of three natural numbers a, b, c with $a^2 + b^2 = c^2$

2. Find all Pythagorean triples of the form $m-1$, $m+1$, $m+3$.

3. Show that there is no Pythagorean triple, a, b, c where all three of a, b, and c are odd.

4. Make a truth table for $(P \to Q) \leftrightarrow (\neg Q \to \neg P)$. Is this a tautology?

5. (a) What is the negation of the statement: $(\exists r \in \mathbb{Q})(r^3 = 2)$
(b) Write both $(\exists r \in \mathbb{Q})(r^3 = 2)$ and its negation as English sentences with no symbols.

6. Prove or give a counterexample: If a, b, c are integers and $a \mid bc$ then $a \mid b$ or $a \mid c$.

7. Prove or give a counterexample: If $a^2 \equiv 0 \pmod 9$, then $a \equiv 0 \pmod 9$.

8. Prove or give a counterexample: If $a^2 \equiv 0 \pmod 3$, then $a \equiv 0 \pmod 3$.

9. Prove or give a counterexample: If n is odd, then $n^2 \equiv 1 \pmod 4$

10. Prove that if $a^2 + a + 1$ is irrational, then so is a.

11. We know that $\sqrt{3}$ is irrational. Use this to show $2\sqrt{3} - 4$ is irrational.

12. Prove: If a, b, c are integers with $a + b + c$ even, then for every integers n the number $an^3 + bn^2 + cn + 1$ is odd.

13. It is true that for any integer a if $3 \mid a^3$, then $3 \mid a$. Use this to prove $\sqrt[3]{3}$ is irrational.

14. Find the sum $1 + 2 + \cdots + 50$.

15. Find the sum $5 - 5(3) + 5(3)^2 - 5(3)^3 + 5(3)^4 - 5(3)^5$.

16. Let a sequence $a_1, a_2, a_3, \ldots$ be defined by

$$a_1 = 1, \qquad a_n = \sqrt{10 + a_{n-1}}.$$

Prove $a_n < 6$ for all n.

17. Use induction to prove if $a \equiv b \pmod{m}$, then for all natural numbers n we have $a^n \equiv b^n$. (You may assume we know that if $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then $ac \equiv bd \pmod{m}$.

18. Draw the Venn diagram for $(A \cap B) \cup C$.

19. Let a be a constant and let $f(x) = xe^{ax}$. Prove that for all positive integers the n-th derivative of f is

$$f^{(n)}(x) = (a^n x + na^{n-1})e^{ax}.$$