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You must show your work to get full credit.

1. Prove if a and b are odd integers, then (a-1)(b-1) is divisible by 4.

Recall at **Pythagorean triple** is a list of three natural numbers a, b, c with $a^2 + b^2 = c^2$. Find all Pythagorean triples of the form m + 1, m + 2.

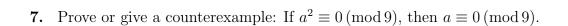
2. Find all Pythagorean triples of the form m-1, m+1, m+3.

3.	Show that	there is no	Pythagorean	triple.	a, b, c where	all three	of a, b .	and c are odd.

4. Make a truth table for
$$(P \to Q) \leftrightarrow (\neg Q \to \neg P)$$
. Is this a tautology?

5. (a) What is the negation of the statement:
$$(\exists r \in \mathbb{Q})(r^3 = 2)$$
 (b) Write both $(\exists r \in \mathbb{Q})(r^3 = 2)$ and its negation as English sentences with no symbols.

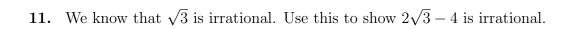
6. Prove or give a counterexample: If a, b, c are integers and $a \mid bc$ then $a \mid b$ or $a \mid c$.



8. Prove or give a counterexample: If
$$a^2 \equiv 0 \pmod{3}$$
, then $a \equiv 0 \pmod{3}$.

9. Prove or give a counterexample: If
$$n$$
 is odd, then $n^2 \equiv 1 \pmod{4}$

10. Prove that if
$$a^2 + a + 1$$
 is irrational, then so is a .



12. Prove: If
$$a, b, c$$
 are integers with $a+b+c$ even, then for every integers n the number an^3+bn^2+cn+1 is odd.

13. It is true that for any integer a if $3 \mid a^3$, then $3 \mid a$. Use this to prove $\sqrt[3]{3}$ is irrational.

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14.	Find	the	sum	- [+2	+	 +	50

15. Find the sum
$$5 - 5(3) + 5(3)^2 - 5(3)^3 + 5(3)^4 - 5(3)^5$$
.

16. Let a sequence
$$a_1, a_2, a_3, \ldots$$
 be defined by

$$a_1 = 1,$$
 $a_n = \sqrt{10 + a_{n-1}}.$

Prove $a_n < 6$ for all n.

17. Use induction to prove if $a \equiv b \pmod{m}$, then for all natural numbers n we have $a^n \equiv b^n$. (You may assume we know that if $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then $ac \equiv bd \pmod{m}$.

18.	Draw the	Venn	diagram	for	$(A \cap$	$B) \sqcup C$	

19. Let a be a constant and let $f(x) = xe^{ax}$. Prove that for all positive integers the n-th derivative of f is

$$f^{(n)}(x) = (a^n x + na^{n-1})e^{ax}.$$