Mathematics 554 Homework.

First let us finish off some of the general results about about integration. Things we would like to be true are that if f and g are integrable on an interval, then so is the product fg and the absolute value. These results are convened in the notes and are the type of thing that are easier to real and work out for yourself rather than listen to someone lecture on them. So read the notes starting at Lemma 3.21 on page 39 and Finishing at Proposition 3.26 on page 40

Problem 1*.* Do Problems 3.14 and 3.15 on page 40 of the notes. □

We have proven the Fundamental Theorem of Calculus in two forms.

Theorem 1 (Fundamental Theorem of Calculus Form 1)**.** *Let $f\colon [a,b] \to \mathbb{R}$ be integrable and define $F\colon [a,b] \to \mathbb{R}$ by*

$$F(x) = \int_a^x f(t)\,dt.$$

Let $x_0 \in (a,b)$ be a point where f is continuous, then F is differentiable at x_0 and

$$F'(x_0) = f(x_0).$$ □

Theorem 2 (Fundamental Theorem of Calculus Form 2)**.** *Let $f\colon [a,b] \to \mathbb{R}$ be a continuous function and let F be an anti-derivative of f (that is $F' = f$ on (a,b). Then*

$$\int_a^b f(x)\,dx = F(b) - F(x)$$ □

There is a slightly better version of the second form of the theorem.

Theorem 3 (Fundamental Theorem of Calculus Form 2, strong version)**.** *Let $f\colon [a,b] \to \mathbb{R}$ be a Riemann integrable function and let F be an anti-derivative of f (that is $F'(x) = f(x)$ for all $x \in (a,b)$. Then*

$$\int_a^b f(x)\,dx = F(b) - F(x)$$

(This differs from the first form in that we only assume that f is integrable, not that it is continuous.)

This is Theorem 4.11 on page 44 of the notes.

One easy application of this is the formula for integration by parts and one application of this, the integral form of the remainder in Taylor's theorem.

Problem 2*.* Do Problems 4.8, 4.9, and 4.10 in the notes. □

Finally we have the change of variable formula (aka u-substitution).

Problem 3*.* Do Problem 4.11 in the notes. □