Mathematics 554 Homework.

Problem 1. We have not computed many specific integrals. One that will show up later in the term is

$$I_n = \int_{-1}^{1} (1-x^2)^n \, dx$$

where n is a positive integer. Here is one way to attach this. Generalize to computing

$$I(m,n) = \int_{-1}^{1} (1-x)^m (1+x)^n \, dx$$

where $m, n \geq 0$ are integers. Then $I_n = I(n,n)$. Show for $m, n \geq 1$ that

$$I(m,n) = \frac{m}{n+1} I(m-1, n+1).$$

and,

$$I(0,N) = \frac{2^{N+1}}{N+1}$$

and use these facts to find I_n. □

Problem 2. Let $f\colon [0,1] \to \mathbb{R}$ be a continuous function with

$$f(x) + f(1-x) = 1.$$

Compute $\displaystyle\int_0^1 f(x)x(1-x)\, dx$. □

Problem 3. In the notes do problems 6.1 to 6.7

Problem 4. For what values of p do the series

$$\sum_{k=2}^{\infty} \frac{1}{k(\ln(k))^p}, \qquad \sum_{k=2}^{\infty} \frac{1}{k\ln(k)(\ln(\ln(k)))^p}$$

converge?

Problem 5. Show that

$$\sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+x} \right)$$

converges of all real numbers $x \notin \{-1, -2, -3, \cdots\}$.