

Mathematics 122 Homework, January 27, 2025.

1. It is known that a radioactive element decays exponentially. Assume that we have start with 23.6 grams of the element, and that after 5 years there are 17.1 grams left.

- (a) Let $A(t)$ be the amount left after t years. Give a formula for $A(t)$.
- (b) What is that half life of the element. That is how long until only half is left.
- (c) How long until there is only 1 gram left.
- (d) What is the average rate of change of $A(t)$ between t and $t + 1$?

2. 1,000\$ is invested at a rate of 20% per year (which is better than you are likely to find in real life.) Let $P(t)$ be the principal after t years.

- (a) Give a formula for $P(t)$.
- (b) How long until the investment doubles.
- (c) How long until the principal reaches one million dollars.

3. Under the right conditions a colony of the bacterium *E. coli* will double in weight every 20 minutes, that is every $1/3$ hour. A bungle *E. coli* weighs 10^{-15} kg. Starting with a single *E. coli* let $W(t)$ be the wight of the colony after t hours.

- (a) Give a formula for $W(t)$.
- (b) The weight of the Earth is 6.0×10^{25} kg. How long until the weight of the colony is equal to the weight of the earth?

4. Here is an example of how even the little we have done with exponentials can be useful. Willard Libby won the 1960 Nobel prize in chemistry for working out how to use the exponential decay of carbon 14, ^{14}C , can be used to date organic samples that are not too old, say 20,000 years or less. If you want more information about the chemistry and physics involved the Wikipedia article on radiocarbon dating is a good source.

The method is based on the fact that ^{14}C has a half life of 5,730 years.

- (a) Give a formula for the percent of ^{14}C left in a sample after t years. Give your answer to 6 decimal places.
- (b) A sample taken from a shroud has only 85.66% of its original ^{14}C left. How old is it?¹

¹This is data from 1988 when radiocarbon dating was used to find the age of the Shroud of Turin.