

*You must show your work to get full credit.*

1. Let  $a > 0$  and define  $f(x) = x^2(a - x)$  on  $0 \leq x \leq a$ .

(a) Find the derivative  $f'(x)$  and simplify.

$f'(x) = \underline{x(2a - 3x)}$

$$\begin{aligned} f'(x) &= (x^2)'(a-x) + x^2(a-x)' \\ &= 2x(a-x) + x^2(-1) \\ &= 2ax - 2x^2 - x^2 \\ &= 2ax - 3x^2 \\ &= x(2a - 3x) \end{aligned}$$

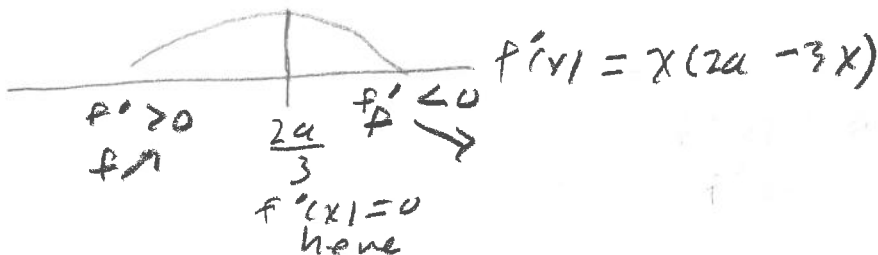
(b) Find the critical points of  $f$ .

The critical points are 0

Solve  
 $f'(x) = x(2a - 3x) = 0$   
 $x = 0$   
 $2a - 3x = 0$   
 $-3x = -2a$   
 $x = \frac{2a}{3}$

(c) What is the maximum and maximizer of  $f$  on  $0 \leq x \leq a$ ?

and The Maximum is  $\frac{4a^2}{27}$  The maximizer is  $\frac{2a}{3}$



$$f\left(\frac{2a}{3}\right) = \left(\frac{2a}{3}\right)^2 \left(a - \frac{2a}{3}\right) = \frac{4a^2}{9} \left(\frac{a}{3}\right) = \frac{4a^3}{27}$$