Mathematics 122

Quiz 5 Name: Key

You must show your work to get full credit.

1. You wish to have \$20,000 in 15 years. How much should you invest now at 5% interest compounded annually achive this?

Let P_0 = amount invested
After t years at 5% simple interest the principal is
$P(t) = P_0(1.05)^t$
we want
$P_0(1.05)^{15} = 20{,}000$
so
$P_0 = \frac{20{,}000}{(1.05)^{15}} = 9620.34$

You should invest \$9,620.34

2. The half life of uranium-238 is 4.5 billion years.

(a) A sample of uranium-238 starts with an amount of A_0 grams. Give a formula for the amoumt left after t billion years.

If $A(t)$ = amount after t years
then $A(t) = A_0 a^t$
$A(4.5) = A_0 a^{4.5} = \frac{1}{2} A_0$ (equation for half life)
so $a^{4.5} = \frac{1}{2} = .5$
$a = (.5)^{\frac{1}{4.5}} = .8572$

$A(t) =$ $A_0 (.8572)^t$

(b) There are rocks from the Slave Lake region in northwest Canada that only have 54% of their original of uranium-238 left. How old are they?

We wish to solve
$A(t) = A_0(.8572)^t = .54 A_0$ (= 54% of A_0)
$(.8572)^t = .54$
$t \ln(.8572) = \ln(.54)$
$t = \frac{\ln(.54)}{\ln(.8572)} = 3.9990$

Their age is 3.999 billion years

Remark: The data in the second problem is close to accurate and there are such rocks from the Slave Lake region. This type of radioactive dating is one of the methods used to compute the age of the earth, which is about 4.5 billion years old.