Mathematics 122

Quiz 6 Name: Key

You must show your work to get full credit.

1. Let $f(x) = x^2 + x$. We will compute the derivative $f'(1)$.

(a) What is average rate of change between 1 and 1.1? Rate is 3.1

$$\frac{\Delta f}{\Delta x} = \frac{f(1.1) - f(1)}{1.1 - 1} = \frac{(1.1)^2 + 1.1 - (1^2 + 1)}{.1}$$

$$= 3.1$$

(b) What is the average rate of change between 1 and 1.01? Rate is 3.01

$$\frac{\Delta f}{\Delta x} = \frac{f(1.01) - f(1)}{1.01 - 1} = \frac{(1.01)^2 + (1.01) - 2}{.01} = 3.01$$

(c) What is the average rate of change between 1 and 1.001? Rate is 3.001

$$\frac{\Delta f}{\Delta x} = \frac{f(1.001) - f(1)}{1.001 - 1} = \frac{(1.001)^2 + (1.001) - 2}{.001} = 3.001$$

(d) At this point you see the pattern, let's use some algebra to make this precise. If $h \neq 0$ what is the average rate of change between 1 and $1 + h$?

Rate is $3 + h$

$$\frac{f(1+h) - f(1)}{1+h-1} = \frac{(1+h)^2 + (1+h) - 2}{h}$$

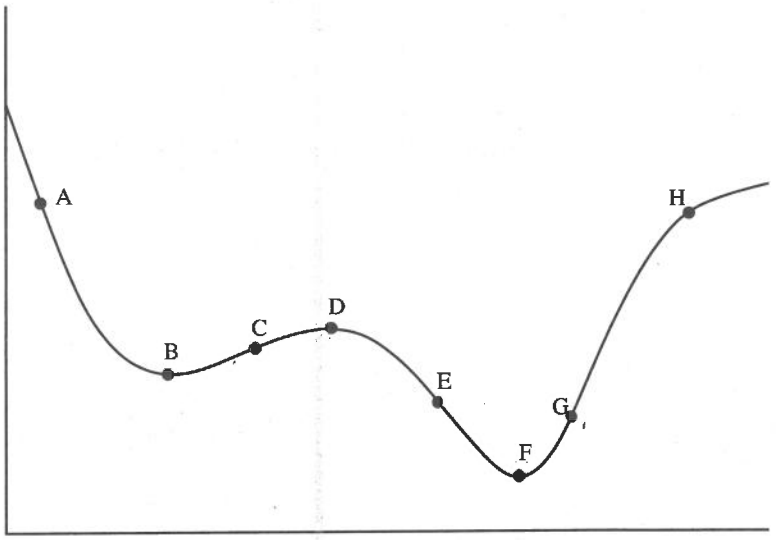
$$= \frac{1 + 2h + h^2 + 1 + h - 2}{h}$$

$$= \frac{3h + h^2}{h} = \frac{h(3+h)}{h} = 3 + h$$

(e) Now let $h = 0$ to get the instantaneous rate of change at $x = 1$.

$f'(1) =$ 3

Let $h = 0$ in $3 + h$ to get 3



2. For the following graph

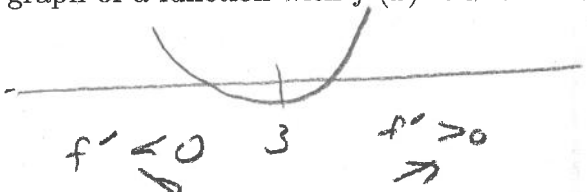
Which of the labeled points have $f'(x) > 0$? C, G, H
(i.e where f increasing)

Which of the labeled points have $f'(x) = 0$? B, D, F

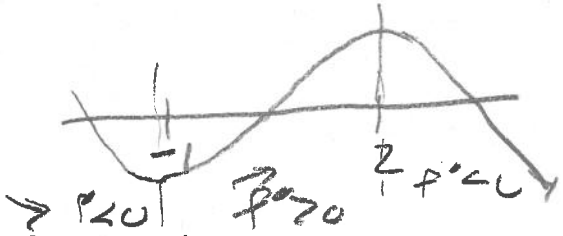
Which of the labeled points have $f'(x) < 0$? A, E
(i.e decreasing)

3. Draw a graph of a function with $f'(x) = 0$

4. Draw a graph of a function with $f'(x) < 0$ for $x < 3$ and $f'(x) > 0$ for $x > 3$.



5. Draw a graph of a function wih $f'(x) < 0$ for $x < -1$ and $x > 2$ and with $f'(x) > 0$ for $-1 < x < 2$.



6. This is a graph of $y = f(x)$. Draw a graph of $y = f'(x)$ on the same axis.