Mathematics 122

Quiz 9 Name: Key

You must show your work to get full credit.

① ② ③

1. The quantities are related by

t	10	12	14	16
P	100	95	90	85

(a) Write some sentences explaining why the relation between t and P is linear showing all the calculations supporting your claim.

slope at ① $= \frac{\Delta P}{\Delta t} = \frac{95-100}{12-10} = -\frac{5}{2} = -2.5$

slope at ② $= \frac{\Delta P}{\Delta t} = \frac{90-95}{14-12} = \frac{-5}{2} = -2.5$

slope at ③ $= \frac{\Delta P}{\Delta t} = \frac{85-90}{16-14} = \frac{-5}{2} = -2.5$

The slopes are constant so it is linear

(b) Write P as a function of t. $P =$ $100 - 2.5(t-10)$

or

$P = -2.5t + 125$

(either form is ok)

$\frac{\Delta P}{\Delta t} = \frac{P-100}{t-10} = -2.5$

$P - 100 = -2.5(t-10)$

$P = 100 - 2.5(t-10)$

$= 100 - 2.5t + 25$

(c) If $P = 87$ what is the value of t? $t =$ 16

$P = 85 = 100 - 2.5(t-10)$

$-15 = -2.5(t-10)$

$t - 10 = \frac{-15}{-2.5} = 6$

$t = 16$

2. Assume the revenue of a vending machine in the t-th week after it is set up is

$$R(t) = \frac{500t^2}{1+t^2} \text{ dollars.}$$

What is the average revenue between $t = 2$ and $t = 4$. Give the units on your answer.

$\frac{\Delta R}{\Delta t} = \frac{R(4) - R(2)}{4-2} = \frac{470.58 - 400}{2} = 35.29$

used calculator

The average rate is 35.29 $\frac{\text{dollars}}{\text{week}}$

① ② ③

3. Let P and t be related by

t	0	1	2	3
P	5.1	6.63	8.619	11.205

(a) Write a couple of sentences explaining why is an exponential relation showing the calculations you used.

Ratio at ① $= \frac{6.63}{5.1} = 1.3$

Ratio at ② $= \frac{8.619}{6.63} = 1.3$

Ratio at ③ $= \frac{11.205}{8.619} = 1.3$

The ration is constant so it is exponential.

(b) What is P as a function of t? $P =$ $5.1(1.3)^t$

$P(t) = P_0 a^t$

$= (\text{initial value})(\text{ratio})^t$

$= 5.1\ (1.3^t)$

(c) What is the value of t when $P = 200$? $t =$ 13.985

$P = 200 = 5.1(1.3)^t$

$(1.3)^t = \frac{200}{5.1}$

$t \ln(1.3) = \ln\left(\frac{200}{5.1}\right)$

$t = \ln(200/5.1)/\ln(1.3) = 13.985$

4. A municipal bond pays 6% annual interest for 30 years. You wish to have \$100,000 in 30 years. How much should you invest in the bonds now to have the \$100,000 in 30 years.

Let P_0 = initial investment.

The amount is \$ 17,411.01

Then after t years the principal is

$P(t) = P_0(1.06)^t$

we want

$P(30) = P_0(1.06)^{30} = 100{,}00$

$P_0 = \frac{100\,000}{(1.06)^{30}} = 17{,}411.01$

5. A radioactive element decays exponentially. It initially weights 12 grams and 10 years latter weights 8.3 grams.

(a) Give a formula for $A(t)$, the amount left after t years.

$A(t) = A_0 a^t = 12 a^t$ $\qquad$ $A(t) =$ $12(.9638)^t$

We Know

$A(10) = 12a^{10} = 8.3$

$a^{10} = \frac{8.3}{12}$

$a = (8.3/12)^{\frac{1}{10}} = .9638$

(b) How long until only %1 of the ordinal amount is left?

We want to solve $\qquad$ $t =$ 281.461 years

$A(t) = \cancel{12}(.9638)^t = .01(\cancel{12})$ (= 1% of original amount)

$(.9638)^t = .01$

$t \ln(.9838) = \ln(.01)$

$t = \ln(.01)/\ln(.9838) = 281.961$

6. Let $g(t) = \dfrac{t^2+3}{t^4+5}$. Use your calculator to compute $g'(1.2)$ to four decimal places.

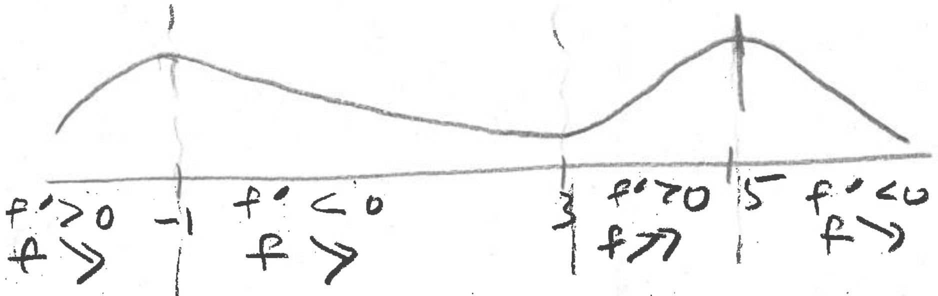
$g'(1.2) =$ $-.2741$

In calculator it is math 8 then

$\frac{d}{dx}((x^2+3)/(x^4+5))\Big|_{x=1.2}$

7. Draw a graph of a function that satisfies

- $f'(x) > 0$ for $x < -1$ or $3 < x < 5$,
- $f'(x) < 0$ for $-1 < x < 3$ or $5 < x$.



8. If $R(10) = 12.3$ and $R'(10) = .45$ estimate the following

$R(10.2) \approx$ $\qquad$ $R(9.9) \approx$ 12.255

$R(a + \Delta x) \approx R(a) + R'(a)\Delta x$

$R(10.2) = R(10 + .2) \approx R(10) + R'(10)(.2)$

$= 12.3 + .45(.2)$

$= 12.39$

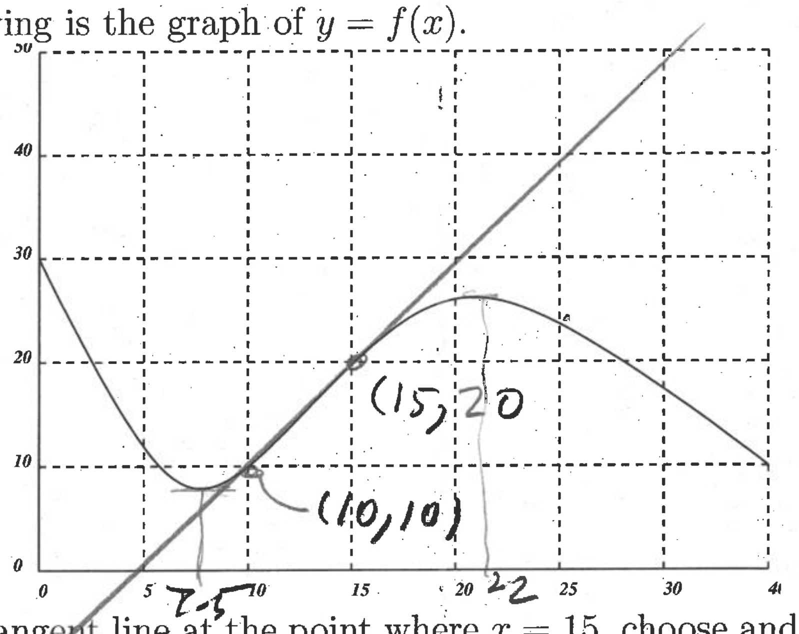
$R(9.9) = R(10 + (-.1))$

$\approx R(10) + R'(10)(-.1)$

$= 12.3 - .45(.1)$

$= 12.255$

9. For the following is the graph of $y = f(x)$.



(a) Draw the tangent line at the point where $x = 15$, choose and label two points on the tangent line and use them to estimate $f'(15)$.

$f'(15)$ = slope of tangent line

$= \frac{\Delta y}{\Delta x} \approx \frac{20-10}{15-10} = \frac{10}{5} = 2$

$f'(15) \approx$ 2

(b) What is the equation of the tangent line to the graph at $x = 15$.

Equation is

$y - y_0 = f'(x_0)(x - x_0)$

in our case $y - 20 = 2(x - 15)$

The equation is $y - 20 = 2(x-15)$ or $y = 2x - 10$ either is ok

(c) At the point where $x = 25$ is the second derivative $f''(25)$ positive or negative?

At $x = 25$ concave down so $f''(25) < 0$

negative

(d) At the point where $x = 25$ is the derivative $f'(25)$ positive or negative?

The function is decreasing so $f' < 0$

negative.

(e) Make an estimate for the values of x where $f'(x) = 0$.

The x values are 7.5, 22